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DBL-003-2015003 Seat No. _____

B. Sc. (Sem. V) (CBCS) (W. E. F.2019) Examination

June - 2022

Mathematics : Paper-07(A)

(Boolean Algebra & Complex Analysis-I)

Faculty Code : 003
Subject Code : 2015003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt any **Five** questions.
(2) Figure to **right** indicate full marks of the question.

1 (A) Answer the following questions : **04**

(1) Define equivalence relation.

(2) State absorption laws for a lattice.

(3) In a lattice $(S_{24,}D)$, find complement of 2, if exists.

(4) Every complemented lattice is distributive. ? (True/ False)

1 (B) Draw Hasse diagram of $(S_{24,}D)$. **02**

1 (C) Let $(L,\leq)$ be a lattice. For any $a,b,c \in L$, prove that $a \leq c$ if and only if $a \oplus (b * c) \leq (a \oplus b) * c$. **03**

1 (D) State and prove De Morgan's laws for a complemented distributive lattice. **05**

2 (A) Answer the following questions : **04**

(1) Define cover of an element.

(2) Let $(L, *, \oplus)$ be a lattice. For any $a, b \in L, a \oplus (a * b) =$ ________.

(3) Let $(L, *, \oplus)$ be a complemented distributive lattice. For any $a, b \in L, (a * b)' =$ ________.

(4) Give an example of a distributive lattice which is not complemented.

2 (B) Let $(L, \leq)$ be a lattice. Prove that for any $a, b \in L, a \leq b$ if and only if $a * b = a$. **02**

2 (C) Let $(L, \leq)$ be a lattice. Prove that for any $a, b, c \in L, a \oplus (b \oplus c) = (a \oplus b) \oplus c$. **03**

2 (D) Prove that every chain is distributive lattice. **05**

3 (A) Answer the following questions. **04**

(1) {1, 2, 5, 10} is a sub-boolean algebra of $(S_{30}, *, \oplus ', 1, 30)$. (True/False)

(2) Define : Atom of a Boolean algebra.

(3) State Stone's representation theorem.

(4) $(x_1 * x_2) \oplus (x_1' \oplus x_2') =$ ________.

3 (B) Prove that in any boolean algebra, complement of every element is unique. **02**

3 (C) Let $(B, *, \oplus, ', 0, 1)$ and $(P, \wedge, \vee, -, \alpha, \beta)$ be two boolean algebra. Prove that if a mapping $f : B \to P$ preserves * and then f is boolean homomorphism. **03**

3 (D) For any boolean algebra $(B, *, \oplus ', 0, 1)$, in usual notations prove that $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2), \forall x_1, x_2 \in B$. **05**

4 (A) Answer the following questions. **04**

(1) Define : Boolean homomorphism.

(2) If a and b are two distinct atoms of Boolean algebra $(B,*,\oplus,',0,1)$ then $a*b=$______.

(3) How many maxterms can be formed using n variables ?

(4) Reduce the expressions $x'y'+x'y$ using K-map.

4 (B) Prove that a nonempty subset S of a boolean algebra $(B,*,\oplus,',0,1)$ is a subboolean algebra if S is closed under * and ' . **02**

4 (C) Let $(B,*,\oplus',0,1)$ be a boolean algebra. Prove that $a \in B$ is an atom if and only if for any $x \in B$, either $a*x=0, a*x=a$. **03**

4 (D) Express $\alpha(x_1,x_2,x_3)=x_1+x_2$ as sum of product canonical form. **05**

5 (A) Answer the following questions. **04**

(1) Determine the points at which the function $f(z)=\dfrac{1}{z^2+1}$ is discontinuous.

(2) Define : Analytic function.

(3) What is imaginary part of $f(z)=\sin z$?

(4) Define : Harmonic function.

5 (B) Using definition of limit, prove that $\lim_{z\to z_0} \operatorname{Im}(z)=\operatorname{Im}(z_0)$. **02**

5 (C) Prove that $f(z)=\bar{z}$ nowhere differentiable. **03**

5 (D) Show that the function. **05**

$$f(z)=\begin{cases} \dfrac{\bar{z}^2}{z}; & z\neq 0; \\ 0; & z=0 \end{cases}$$

Is not differentiable at origin although Cauchy-Riemann equation are satisfied at origin.

6 (A) Answer the following questions. **04**

(1) What is the real part of $f(z) = e^x$?

(2) Define : Function of complex variable.

(3) Define : Entire function.

(4) Limit of function of complex variables need not be unique. (True / False)

6 (B) Using definition of limit, prove that $\lim_{z \to z_0} az + b = az_0 + b$, where a and b are complex. **02**

6 (C) State and prove necessary condition for a function to be analytic. **03**

6 (D) Show that $u(x, y) = e^x \cos y$ *and* $y^3 - 3x^2 y$ are harmonic functions. **05**

7 (A) Answer the following questions. **04**

(1) State Cauchy-Riemaan equations in polar form.

(2) Show that $f(z) = xy + iy$ is nowhere analytic.

(3) Define : Simply connected domain.

(4) If C represents the unit circle then $\int_c \frac{1}{z-3} dz =$ _______.

7 (B) If $f(z) = u + iu$ is analytic function and u is constant, then prove that $f(z)^3$ is constant. **02**

7 (C) If $f(z) = u + iu$ is analytic function, then prove that u and v satisties Laplace equation. **03**

7 (D) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$ **05**

8 (A) Answer the following questions. **04**

(1) State Cauchy-Riemann equations in cartesian form.

(2) Define : Harmonic conjugete.

(3) State Cauchy - Goursat Theorem.

(4) If C is $|z|=1$, then $\int_C \frac{1}{z} dz =$_______.

8 (B) Evaluate $\int_C \frac{1}{z-z_0} dz$,where C is $|z-z_0|=r$. **02**

8 (C) If $f(z)=u+iv$ is analytic function then prove that $u(x,y)=C_1$ and $v(x,y)=C_2$ represent the family of orthogonal curves (C_1 and C_2 are constants). **03**

8 (D) State and prove Cauchy's integral formula. **05**

9 (A) Answer the following questions. **04**

(1) Write Cauchy's extended integral formula for derivatives of an analytic function.

(2) cos z is bounded function. (True/False)

(3) State Maximum modulus principle.

(4) State fundamental theorem of algebra.

9 (B) If C : $|z|=1$, then evaluate $\int_C \frac{\sin z}{\left(z-\frac{\pi}{6}\right)^3} dz$ **02**

9 (C) State and prove Cauchy's inequality. **03**

9 (D) State and prove Morera's theorem. **05**

10 (A) Answer the following questions : **04**

(1) What is the number of roots in $\mathbb{C}$ for the polynomial $z^5 + 1 = 0$?

(2) An entire bounded function is always_______.

(3) Which theorem is the converse of Cauchy-Gourast theorem ?

(4) If $f(z)$ is entire function and real part of $f(z)$ is bounded, then f is constant (True / False)

10 (B) Evaluate $\int_C \frac{\cosh z}{z^3} dz$, where $C : |z| = 1$. **02**

10 (C) State and prove Liouville's theorem. **03**

10 (D) State and prove Morera's theorem. **05**